Extraction of Internal Tidal Currents and Reconstruction of Full-Depth Tidal Currents from Mooring Observations

AN-ZHOU CAO, BING-TIAN LI, AND XIAN-QING LV

Key Laboratory of Physical Oceanography, Ocean University of China, Ministry of Education, Qingdao, China

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ABSTRACT

To obtain internal tidal currents and full-depth tidal currents from limited mooring observations, a method is put forward combining harmonic analysis and modal decomposition. Harmonic analysis is used to separate tidal currents of different constituents, and modal decomposition is used to calculate full-depth tidal currents of each mode. By adding the barotropic tidal currents to all the baroclinic ones, the full-depth tidal currents of each constituent are reconstructed. The feasibility and accuracy of the proposed method is tested by twin experiments. Then, the method is used to extract tidal currents of each mode and to reconstruct full-depth tidal currents for M2 and K1 from a 3-month-long time series of acoustic Doppler current data observed at a station in the northern South China Sea. Results indicate that the total kinetic energy (KE) of M2 is 25% larger than that of K1. For M2, the first baroclinic mode is the dominant one, followed by the barotropic one, and the sum of these modes accounts for more than 90% of the total M2 KE. Tidal constituent K1 is dominated by the barotropic mode, which accounts for more than 90% of the total K1 KE.

1. Introduction

Internal tides are internal waves with tidal frequency that are generated in the stratified ocean by barotropic tidal currents flowing over varying topographies, such as midocean ridges, seamounts, continental shelves, and slopes. In the tides-to-mixing cascade, internal tides play an important role in dissipating surface tides (Munk 1997; Egbert and Ray 2000) and enhancing oceanic mixing (Niw a and Hibiya 2001; Rudnick et al. 2003) and, consequently, contribute to large-scale ocean circulations.

One direct and important approach to investigate internal tides is to analyze mooring observations containing currents measured by acoustic Doppler current profilers (ADCP s) and temperature by thermometers. Near the Hawaiian Ridge and Luzon Strait, which are two typical sources of internal tides, internal tides have been detected and studied by many scholars. Rudnick et al. (2003) mentioned that the peak-to-peak amplitude of semidiurnal internal tides near the Hawaiian Ridge could reach 300 m by analyzing mooring temperature data. Chavanne et al. (2010) investigated the semidiurnal internal tides near Hawaii by comparing the observed currents with numerical model predictions and analytical model results. Zilberman et al. (2011) studied the incoherent nature of M2 internal tides at the Hawaiian Ridge by using the current, temperature, and conductivity data, and found that the energy conversion near the ridge crest varied considerably by a factor of 2 (0.5–1.1 W m⁻²). By analyzing 6-month records of current profiles, the nonlinear energy transfer from M2 internal tides to diurnal waves in the Kauai Channel (Hawaii) was detected by Chou et al. (2014). For the internal tides near the Luzon Strait, Alford et al. (2011) investigated their generation, propagation, and dissipation using two profiling moorings containing ADCPs and CTDs. Results indicated that the peak-to-peak baroclinic velocity and vertical displacement exceeded 2 m s⁻¹ and 300 m, respectively, and that the energy flux exceeded 60 kW m⁻¹ at the spring tide. Klymak et al. (2011) studied the breaking and scattering of internal tides on the continental shelf of the South China Sea (SCS) using mooring data and calculated the energy flux and turbulence dissipation. Using current data obtained from mooring ADCPs, Xu et al. (2011, 2013, 2014) investigated the seasonal variation and multimodal structure of internal tides at three stations in the SCS. In

Corresponding author address: Xian-Qing Lv, Key Laboratory of Physical Oceanography, Ocean University of China, Ministry of Education, 238 Songling Road, Qingdao 266100, China. E-mail: xqinglv@ouc.edu.cn

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addition, Katsumata et al. (2010), Ross et al. (2014), Tanaka et al. (2014), and Terker et al. (2014) researched the internal tides on the Timor shelf, in the Patagonian fjord, in the Bussol Strait, and on the California continent margin using mooring observations, respectively.

There still exists a problem in the acquisition of full-depth observations because of the water depth at the mooring station, the number and measuring range limitation of ADCPs, etc. Some scholars used linear interpolation (extrapolation) to obtain the missing data beyond the measuring range (Katsumata et al. 2010). However, linear interpolation (extrapolation) seems not to be a good method for the lack of physical explanations. Webb and Pond (1986) put forward a method to extract internal tides in the Knight Inlet using modal decomposition. As a method to separate motions of each mode based on the eigenfunctions of an eigenvalue problem, modal decomposition is widely used in data analysis (Kelly et al. 2010; Buijsman et al. 2010; Klymak et al. 2011; Ma et al. 2013), theoretical (Griffiths and Grimshaw 2007) and numerical models (Heaps 1983), and it performs as well in the estimation of modal motions as the empirical orthogonal function, ridge regression, and optimal estimate (Smith et al. 1985).

According to the method in Webb and Pond (1986), up-inlet and down-inlet internal tides in the Knight Inlet were extracted from observations. In the method, observations containing density (temperature and salinity), currents, and barotropic tides are essential because equations of horizontal currents and density perturbations need to be solved simultaneously. However, it is difficult to have all these observations for general moorings, especially in the deep ocean. In addition, criteria about the normal mode fitting are not very clear in Webb and Pond (1986). So, in this study, a general method is put forward to extract tidal currents of each mode and to reconstruct full-depth tidal currents from limited mooring observations by using harmonic analysis and modal decomposition. In the proposed method, only the current data are essential [stratification can be calculated using the World Ocean Atlas 2005 (WOA05) data] and the determination of normal mode fitting is discussed in detail in this study. The paper is organized as follows. Section 2 introduces the mooring data and the method to reconstruct full-depth tidal currents. In section 3, twin experiments (TEs) are carried out to test the feasibility of the proposed method. Based on the proposed method and a 3-month-long time series of acoustic Doppler current data, tidal characteristics of M\(_2\) and K\(_1\) at a station in the northern SCS are investigated in section 4. Finally, the paper is summarized in section 5.

2. Data and methodology

a. Data

In this study, a 3-month-long (from 1 March 2014 to 31 May 2014) time series of current data obtained from two 75-kHz ADCPs located at a station (21.1°N, 117.9°E) in the northern SCS is used. Water depth at this station is 980 m. Two ADCPs were both positioned at 400-m depth, of which one was uplooking to capture currents from 50 to 370 m and the other was down-looking to capture currents from 430 to 880 m. Current data were recorded with precision of 5 × 10\(^{-3}\) ms\(^{-1}\) and a time interval of 1 h. In addition, temperature between 100- and 400-m depth was also measured by thermistor chains. The observed currents and temperature are shown in Fig. 1. As seen, the observed currents are not full depth due to the measuring range limitation and vertical movement of each ADCP.

b. Methodology

To obtain full-depth tidal currents and internal tidal currents of each mode for the principal constituents from limited observations, a method is put forward combining harmonic analysis and modal decomposition. Based on the least squares method, harmonic analysis (Fang et al. 1999) is carried out to calculate harmonic constants of tidal currents of the principal constituents. Using these harmonic constants, time series of currents of each constituent are calculated and then used in modal decomposition to calculate full-depth tidal currents of each mode (containing both the barotropic and baroclinic modes). Adding the barotropic tidal currents to all the baroclinic ones, the full-depth tidal currents of each constituent are reconstructed.

Modal decomposition used in this study is similar to that in Griffiths and Grimshaw (2007), which is described as follows for convenience. In general, horizontal tidal currents can be decomposed as

\[
\begin{align*}
\mathbf{u}(z, t) &= \sum_{m=0}^{\infty} \mathbf{u}_m(z, t) = \sum_{m=0}^{\infty} \left[ \mathbf{U}_m(t) \cdot \partial \phi_m(z) / \partial z \right], \\
\mathbf{v}(z, t) &= \sum_{m=0}^{\infty} \mathbf{v}_m(z, t) = \sum_{m=0}^{\infty} \left[ \mathbf{V}_m(t) \cdot \partial \phi_m(z) / \partial z \right],
\end{align*}
\]

(1)
where \( u \) and \( v \) represents tidal current components corresponding to one constituent (e.g., M\(_2\) or K\(_1\)); \( u_m \) and \( v_m \) are tidal current components of the \( m \)th mode (\( m = 0 \) represents the barotropic mode and \( m > 0 \) the baroclinic modes); \( U_m \) and \( V_m \) are coefficients corresponding to \( u_m \) and \( v_m \), respectively; and \( \phi_m \) are eigenfunctions of the eigenvalue problem for wave speed \( c \):

\[
\begin{align*}
\frac{\partial^2 \phi}{\partial z^2} + \frac{N^2}{c^2} \phi &= 0 \\
\phi &= 0 \text{ at } z = 0 \\
\phi &= 0 \text{ at } z = h,
\end{align*}
\]

where \( N \) is the buoyancy frequency and \( h \) is the water depth. Typically, there will be an infinite number of eigenvalues \( c_m, m = 0, 1, 2, \ldots \), with corresponding eigenfunctions \( \phi_m \), that form a complete set for a certain class of functions on \( 0 \leq z \leq h \). A finite number (i.e., \( m = 1, 2, \ldots, M \)), rather than an infinite number, of eigenvalues and eigenfunction is always used. In addition, the eigenfunctions should satisfy an orthogonality condition, that is,

\[
\int_0^h \frac{\partial \phi_m}{\partial z} \frac{\partial \phi_n}{\partial z} \, dz = \begin{cases} 0 & n \neq m \\ 1 & n = m \end{cases}.
\]

According to Griffiths and Grimshaw (2007), for the barotropic mode,

\[
c_0 = \sqrt{gh} \quad \text{and} \quad \phi_0 = 1 + z/h,
\]

where \( g \) is the acceleration due to gravity. Using the Wentzel–Kramers–Brillouin (WKB) approximation, the baroclinic modes can be calculated from Eq. (2) by taking

\[
I = \frac{\pi^2}{2h},
\]

and the corresponding eigenvalues and eigenfunctions are

\[
c_m = c_m^{\text{WKB}} = \frac{Nh}{m\pi} \quad \text{and} \quad \phi_m = \phi_m^{\text{WKB}} = \frac{(-1)^m}{m} \sqrt{\frac{N}{Nh}} \int_0^h N(z') \, dz',
\]

where

\[
N = \frac{1}{h} \int_0^h N(z) \, dz
\]

is the depth-averaged buoyancy frequency. Then, the normal modes in Eq. (1) can be easily calculated as

\[
\frac{\partial \phi_0}{\partial z} = \frac{1}{h} \quad \text{and} \quad \frac{\partial \phi_m}{\partial z} = \frac{(-1)^m \pi}{h} \sqrt{\frac{N}{Nh}} \int_0^h N(z') \, dz'.
\]

The buoyancy frequency can be derived from observations or World Ocean Atlas 2005 (WOA05) data and then used to calculate the normal modes according to

Fig. 1. Observed current components (a) \( u \) and (b) \( v \) (m s\(^{-1}\)), positive when pointing east and north, respectively; and (c) temperature (°C).
Eq. (8). Using these normal modes and time series of tidal current components obtained from observations, $U_m$ and $V_m$ can be calculated with the least squares method according to Eq. (1). Taking current component $u$ as an example, a detailed procedure is shown as follows:

$$
\min \ L = \sum_{j=1}^{J} \left\{ u(z_j, t_0) - \sum_{m=0}^{M} \left[ U_m(t_0) \cdot \partial \phi_m(z_j)/\partial z \right] \right\}^2,
$$

where $t_0$ represents an arbitrary moment during the observation; $z_j, j = 1, 2, \ldots, J$ represent the observing depths; and $M$ is the total number of normal modes used in modal decomposition. It should be noted that $M$ may be different for different tidal constituents. To solve Eq. (9), there will be

$$
\begin{align*}
\frac{\partial L}{\partial U_0} &= 0 \\
\frac{\partial L}{\partial U_1} &= 0 \\
& \quad \vdots \\
\frac{\partial L}{\partial U_M} &= 0
\end{align*}
$$

that is,

$$
\begin{align*}
U_0 &\cdot \sum_{j=1}^{J} (\phi_0^j)^2 + U_1 &\cdot \sum_{j=1}^{J} (\phi_0^j \phi_1^j) + \cdots + U_M &\cdot \sum_{j=1}^{J} (\phi_0^j \phi_M^j) = \sum_{j=1}^{J} (u \phi_0^j) \\
U_0 &\cdot \sum_{j=1}^{J} (\phi_1^j)^2 + U_1 &\cdot \sum_{j=1}^{J} (\phi_1^j \phi_1^j) + \cdots + U_M &\cdot \sum_{j=1}^{J} (\phi_1^j \phi_M^j) = \sum_{j=1}^{J} (u \phi_1^j) \\
& \quad \vdots \\
U_0 &\cdot \sum_{j=1}^{J} (\phi_M^j)^2 + U_1 &\cdot \sum_{j=1}^{J} (\phi_M^j \phi_1^j) + \cdots + U_M &\cdot \sum_{j=1}^{J} (\phi_M^j \phi_M^j) = \sum_{j=1}^{J} (u \phi_M^j)
\end{align*}
$$

By solving Eq. (11), $U_m, m = 0, 1, \ldots, M$, can be calculated. Similarly, $V_m, m = 0, 1, \ldots, M$, can be calculated. Thereafter, full-depth barotropic and baroclinic tidal currents can be obtained according to Eq. (1). By adding the barotropic tidal currents to all the baroclinic ones, the full-depth tidal currents are reconstructed.

3. Twin experiments

In this section, TEs are carried out to test the feasibility of the proposed method. For simplicity, only $M_2$ is considered in the TEs. The process of TEs is designed as follows. 1) Based on the normal modes at the mooring station, full-depth barotropic and baroclinic tidal currents are prescribed, which are then used to compose a 3-month-long time series of $M_2$ tidal currents. The composed $M_2$ tidal currents are regarded as the prescribed “observations.” 2) According to the proposed method, full-depth $M_2$ tidal currents are reconstructed from these “observations.” It should be noted that in order to be consistent with reality, only the observations
in the ADCP measuring range (shown in Fig. 4a) are used. 3) By calculating the difference between the prescribed and reconstructed tidal currents, the method can be assessed.

Before TEs are carried out, it is necessary to analyze the stratification and to calculate the normal modes at the mooring station. Figure 2 shows the spring temperature, salinity, and density from WOA05 data (time averaged from March to May). The observed temperature is also plotted in Fig. 2a for comparison (to remove the influence of the mesoscale eddy, which occurred in April, shown in Fig. 1c, the time-averaged temperature in May is regarded as the spring temperature for observations). The observed temperature shows good agreement with that from WOA05 data, so the density obtained from WOA05 data can be regarded as the real value and used to calculate the buoyancy frequency (Fig. 3a). According to Eqs. (6) and (8), $\varphi_m$ and $\partial \varphi_m / \partial z$ are calculated, which are shown in Figs. 3b and 3c, respectively.

a. TE1: Feasibility of the method

Based on the normal modes shown in Fig. 3c, one barotropic mode and three baroclinic modes are prescribed in TE1 to compose $M_2$ tidal currents, which are shown in Fig. 4. Using the proposed method and limited observations, barotropic and baroclinic tidal currents of each mode are calculated and full-depth $M_2$ tidal currents are reconstructed. Because the total number of normal modes ($M$) is important in the proposed method, different values of $M$ are used in TE1. Figure 5 displays the reconstructed results and corresponding modes when

![Fig. 3. (a) Buoyancy frequency, and parts of (b) $\varphi_m$ and (c) $\partial \varphi_m / \partial z$ at the mooring station.](image)

![Fig. 4. Prescribed (b) barotropic and (c)–(e) baroclinic modes, and (a) composed $M_2$ tidal currents in TE1. Black line in each subfigure represents the instantaneous currents at 0100 local time (UTC+8) 1 Mar 2014. Shadows in (a) denote that the data at the corresponding depths are beyond the ADCP measuring range and are not used in the proposed method.](image)
$M = 1$ and 2. It is obvious that both reconstructed $M_2$ tidal currents and corresponding modes have a large difference from the prescribed ones, suggesting that when $M$ is smaller than the number of prescribed modes, the prescribed motion cannot be reconstructed accurately. However, when $M \geq 3$, the prescribed motion can be reconstructed accurately, indicating that it is possible to reconstruct the prescribed tidal currents using limited observations. Table 1 lists the mean absolute errors (MAEs) between the prescribed and reconstructed $M_2$ tidal currents in the ADCP measuring range corresponding to different values of $M$. As can be seen, more modes lead to smaller differences until $M$ is equal to or larger than the number of prescribed modes. Then, it is concluded that the proposed method is feasible when sufficient modes are used.

**b. TE2: Effect of measurement errors**

Because ADCPs have measurement errors in the real observations, the effect of measurement errors should be considered. Based on the prescribed modes in TE1 (Fig. 4), random errors are added to the composed $M_2$ tidal currents (Fig. 6). The maximum errors in amplitudes of $u$ and $v$ are 0.5 cm s$^{-1}$, which is comparable to the precision of the ADCP. In TE2, different values of $M$ are used (from 1 to 10). The reconstructed results and differences between prescribed and reconstructed tidal currents corresponding to different values of $M$ are displayed in Fig. 7 and Table 2, respectively. Similar to results in TE1, the reconstructed $M_2$ tidal currents show large differences from the prescribed ones both inside and outside the ADCP measuring range when $M < 3$. With larger $M$ ($M = 3$, 4, 5, and 6), the prescribed $M_2$ tidal currents are reconstructed more accurately.

---

**Table 1. Differences between the prescribed and reconstructed $M_2$ tidal currents in the ADCP measuring range in TE1.**

<table>
<thead>
<tr>
<th>No. of modes ($M$)</th>
<th>MAE of amplitude of $u$ (cm s$^{-1}$)</th>
<th>MAE of phase of $u$ (°)</th>
<th>MAE of amplitude of $v$ (cm s$^{-1}$)</th>
<th>MAE of phase of $v$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.2</td>
<td>5.8</td>
<td>1.0</td>
<td>9.9</td>
</tr>
<tr>
<td>2</td>
<td>0.4</td>
<td>2.2</td>
<td>0.2</td>
<td>3.3</td>
</tr>
<tr>
<td>≥3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Tidal currents can be reconstructed, suggesting that the method is feasible when the appropriate mode number is used in the proposed method, although measurement errors exist in the observations. Indeed, when $M = 3, 4, 5, \text{ and } 6$, the extracted barotropic and first three baroclinic modes are almost the same as the prescribed ones, and the extra modes (e.g., mode 4 when $M = 4$, mode 4 and mode 5 when $M = 5$) are too weak to be detected. However, when more modes are used ($M = 7, 8, 9, \text{ and } 10$), unreasonable reconstructed results appear beyond the ADCP measuring range, especially in the upper 100 m (Figs. 7g–j). To study the cause of the
unreasonable results, Fig. 8 displays the extracted barotropic and baroclinic modes with $M = 10$. As can be seen, the extracted results of modes 0–3 show large differences from the prescribed ones and unreasonable results of modes 4–8 appear. Comparing these results with those in TE1, it can be concluded that the measurement errors cause the unreasonable results. To investigate how the measurement errors cause the unreasonable results, attention should be refocused on the proposed method. Actually, the appearance of unreasonable results (e.g., Fig. 8) implies the nonuniqueness of the solution of Eq. (11), which might be attributed to the least squares method and/or the nonorthogonality of normal modes (due to limited observing depths) in the proposed method. However, according to results in Table 2, differences between the prescribed and reconstructed tidal currents within the ADCP measuring range almost keep invariant when $M > 3$, suggesting that the least squares method is not the reason. Then, it can be deduced that the existence of measurement errors and the nonorthogonality of normal modes lead to the nonuniqueness of the solution of Eq. (11) and thus to unreasonable reconstructed results.

### Table 2. Differences between the prescribed and reconstructed M$_2$ tidal currents in the ADCP measuring range in TE2.

<table>
<thead>
<tr>
<th>No. of modes ($M$)</th>
<th>MAE of amplitude of $u$ (cm s$^{-1}$)</th>
<th>MAE of phase of $u$ (°)</th>
<th>MAE of amplitude of $v$ (cm s$^{-1}$)</th>
<th>MAE of phase of $v$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.19</td>
<td>5.89</td>
<td>1.00</td>
<td>10.21</td>
</tr>
<tr>
<td>2</td>
<td>0.44</td>
<td>2.51</td>
<td>0.22</td>
<td>3.92</td>
</tr>
<tr>
<td>3</td>
<td>0.16</td>
<td>1.14</td>
<td>0.16</td>
<td>2.03</td>
</tr>
<tr>
<td>4</td>
<td>0.16</td>
<td>1.13</td>
<td>0.16</td>
<td>2.05</td>
</tr>
<tr>
<td>5</td>
<td>0.15</td>
<td>1.13</td>
<td>0.16</td>
<td>2.06</td>
</tr>
<tr>
<td>6</td>
<td>0.16</td>
<td>1.11</td>
<td>0.16</td>
<td>2.05</td>
</tr>
<tr>
<td>7</td>
<td>0.16</td>
<td>1.09</td>
<td>0.16</td>
<td>2.03</td>
</tr>
<tr>
<td>8</td>
<td>0.15</td>
<td>1.08</td>
<td>0.16</td>
<td>2.03</td>
</tr>
<tr>
<td>9</td>
<td>0.15</td>
<td>1.09</td>
<td>0.16</td>
<td>2.02</td>
</tr>
<tr>
<td>10</td>
<td>0.15</td>
<td>1.09</td>
<td>0.16</td>
<td>2.03</td>
</tr>
</tbody>
</table>

**Fig. 8.** Extracted (a) barotropic and (b)–(k) baroclinic modes in TE2 when $M = 10$. 
when more modes are used in the proposed method. However, the nonorthogonality of normal modes and measurement errors would always exist because the aim is to reconstruct full-depth tidal currents from limited observations. Therefore, when dealing with real mooring observations, an alternative way is to try as many modes as possible until the unreasonable results appear.

c. Comparison with linear interpolation (extrapolation)

In this subsection, the proposed method is compared with linear interpolation (extrapolation). Based on the observations in TE2, linear interpolation (extrapolation) is used to reconstruct the missing data of tidal currents at 375–475-m (0–80 m) depth, which correspond to the middle (highest) shadow in Fig. 6a. The M2 tidal currents reconstructed with the proposed method are the results of TE2 with $M = 3$ (Fig. 7c). Tables 3 and 4 show the differences between the prescribed and reconstructed tidal currents at 375–475- and 0–80-m depths, respectively. Although both the proposed method and linear interpolation can reconstruct the missing data with small MAEs at 375–475-m depth, where the prescribed tidal currents vary slightly, all the MAEs caused by the proposed method are smaller than those by linear interpolation. However, at 0–80-m depth, where the prescribed tidal currents vary considerably, the MAEs corresponding to linear extrapolation are much larger than those corresponding to the proposed method, especially for the MAEs of amplitudes of $u$ and $v$, suggesting that the proposed method is better than linear interpolation (extrapolation).

From all the results in section 3, it can be concluded that the proposed method is feasible in extracting tidal currents of each mode and reconstructing full-depth tidal currents from limited observations, and that it is better than linear interpolation (extrapolation).

4. Reconstructed results and tidal characteristics at the mooring station

Based on the proposed method and the 3-month-long time series of current data, one barotropic mode and three baroclinic modes are extracted for M2, and one barotropic mode and two baroclinic modes for K1 at the mooring station. Figure 9 displays tidal current ellipses of these modes, as well as the observed and reconstructed M2 and K1, which are the dominant semidiurnal and diurnal tides at the mooring station, respectively. Tidal constituent M2 is dominated by the first baroclinic mode, while K1 is dominated by the barotropic mode. With the increase of the baroclinic modal number, the corresponding motion becomes weaker gradually for both M2 and K1, which is consistent with the fact that the high-mode baroclinic motions are easy to dissipate. To obtain quantitative features, the period-averaged kinetic energy (KE) of each mode is calculated as

$$E_m = \frac{1}{4} \int_0^h \rho (A_{um}^2 + A_{vm}^2) \, dz,$$

where $\rho$ is the density, and $A_{um}$ and $A_{vm}$ are the amplitudes of the $m$th mode of $u$ and $v$, respectively. Table 5 shows the KE of each mode for M2 and K1 and their corresponding proportions. On the whole, the total KE of M2 is 25% larger than that of K1. For M2, the first baroclinic mode is dominant, followed by the barotropic mode, and the sum of these two modes can account for more than 90% of the total M2 KE. The third baroclinic mode only accounts for 1.3% of the total M2 KE, which can be neglected compared with other modes. For K1, the proportion of the barotropic mode is larger than 90%, which is almost equal to the proportion of the first two modes for M2. As the baroclinic modes of K1 are weak, their major contribution is made to the vertical variation of the current phase, which is shown in Fig. 9h.

| Table 3. Differences between the prescribed and reconstructed M2 tidal currents at 375–475-m depth corresponding to the proposed method and linear interpolation. |
|---|---|---|---|---|
| No. of modes ($M$) | MAE of amplitude of $u$ (cm s$^{-1}$) | MAE of phase of $u$ ($^\circ$) | MAE of amplitude of $v$ (cm s$^{-1}$) | MAE of phase of $v$ ($^\circ$) |
| Proposed method | 0.14 | 1.26 | 0.19 | 3.00 |
| Interpolation | 0.27 | 2.97 | 0.20 | 4.05 |

| Table 4. Differences between the prescribed and reconstructed M2 tidal currents at 0–80-m depth corresponding to the proposed method and linear extrapolation. |
|---|---|---|---|---|
| No. of modes ($M$) | MAE of amplitude of $u$ (cm s$^{-1}$) | MAE of phase of $u$ ($^\circ$) | MAE of amplitude of $v$ (cm s$^{-1}$) | MAE of phase of $v$ ($^\circ$) |
| Proposed method | 0.28 | 0.69 | 0.27 | 1.19 |
| Extrapolation | 4.08 | 22.98 | 2.36 | 1.48 |
5. Summary

In this study, a method is put forward to extract internal tidal currents and to reconstruct full-depth tidal currents from limited observations by using harmonic analysis and modal decomposition. Harmonic analysis is used to separate tidal currents of different constituents, and modal decomposition is used to calculate the corresponding full-depth barotropic and baroclinic tidal currents using the least squares method. Adding the barotropic tidal currents to all the baroclinic ones, the full-depth tidal currents of each constituent are reconstructed.

The feasibility and accuracy of the proposed method is tested by twin experiments. Results indicate that the prescribed motion can be reconstructed by adopting the appropriate number of modes in the proposed method. Because of the existence of measurement errors and the nonorthogonality of normal modes, which is due to limited observing depths, too many modes used in the method may cause unreasonable results. Besides, the proposed method is better than linear interpolation (extrapolation) in reconstructing the missing data of tidal currents.

Finally, the proposed method is used to extract the tidal currents and to reconstruct the full-depth tidal currents for \( \text{M}_2 \) and \( \text{K}_1 \) from the 3-month-long time series of acoustic Doppler current data observed at a station in the northern SCS. Results denote that the total KE of \( \text{M}_2 \) is 25% larger than that of \( \text{K}_1 \). For \( \text{M}_2 \), the first baroclinic mode is dominant and is followed by the barotropic mode, and the sum of these two modes accounts for more than 90% of the total \( \text{M}_2 \) KE. For \( \text{K}_1 \), the proportion of the barotropic mode is larger than 90%, which is almost equal to the proportion of the first two modes for \( \text{M}_2 \). As baroclinic modes of \( \text{K}_1 \) are weak, they contribute more to the vertical variation of the current phase rather than to amplitude.

![Observed and reconstructed tidal currents](image)

**TABLE 5.** KE (J m\(^{-2}\)) of each mode and corresponding proportions for \( \text{M}_2 \) and \( \text{K}_1 \).

<table>
<thead>
<tr>
<th>Constituent</th>
<th>Mode 0</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{M}_2 )</td>
<td>881 (27.3%)</td>
<td>2102 (65.0%)</td>
<td>206 (6.4%)</td>
<td>43 (1.3%)</td>
<td>3232</td>
</tr>
<tr>
<td>( \text{K}_1 )</td>
<td>2324 (90.3%)</td>
<td>191 (7.4%)</td>
<td>60 (2.3%)</td>
<td>—</td>
<td>2575</td>
</tr>
</tbody>
</table>
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REFERENCES


