Harmonic Analysis in the Simulation of Multiple Constituents: Determination of the Optimum Length of Time Series

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ABSTRACT

To investigate the optimum length of time series (TS) for harmonic analysis (HA) in the simulation of multiple constituents, a two-dimensional tidal model is used to simulate the M_2, S_2, K_1, and O_1 constituents in the Bohai and Yellow Seas. By analyzing the HA results of several nonoverlapping TS of the same length, which varies from 15 to 365 days, a field-average deviation of HA results is calculated. A deviation that is sufficiently small means that HA results are independent of the choice of TS, and the corresponding TS length is regarded as the optimum. Results indicate that the range of 180–195 days is the optimum length of TS for HA in the simulation of the four principal constituents. To investigate what determines the optimum length, experiments with different computed area and model settings are carried out. Results indicate that the optimum length is independent of advection, nodal corrections, and computed area, and only depends on bottom friction. Nonlinear bottom friction results in the appearance of higher harmonics and explains why the optimum length of TS for HA is 180–195 days.

1. Introduction

Nowadays, there seems to be no secrets about the two-dimensional tides, as many numerical models (Zu et al. 2008; Rosenfeld et al. 2009; Wijeratne et al. 2012; Wang et al. 2013; and reference therein) and inverse models (Egbert and Erofeeva 2002; Zhang et al. 2011; Cao et al. 2012; and references therein) have been applied to the simulation of global and regional tides. However, for the simultaneous simulation of multiple constituents, no conclusion has been drawn about the optimum length of time series (TS) for harmonic analysis (HA). Cummins and Oey (1997) applied the Princeton Ocean Model to the simulation of the semidiurnal and diurnal barotropic tides off north British Columbia. In their simulation, the model ran for 34 days and hourly results of the last 29 days were subject to HA. Kang et al. (1998) investigated five major constituents (M_2, S_2, K_1, O_1 and N_2) and the shallow-water constituents M_4 and MS_4 in the Yellow and East China Seas. After an initial warming run, simulated results of a continuous 28-day run were subject to HA. Fang et al. (1999) carried out the simulation of the four principal constituents (M_2, S_2, K_1, and O_1) in the South China Sea (SCS), the Gulf of Tokin, and the Gulf of Thailand for 375 days. The initial 10-day-long TS was discarded and the following year-long TS of surface elevation and horizontal velocities were analyzed to yield harmonic constants (HCs). For the M_2, S_2, K_1, and O_1 constituents on the West Florida shelf, He and Weisberg (2002) used an initial spinup period of five inertial cycles (about 5 days) to suppress transients and data of the subsequent 30 days for HA. Zu et al. (2008) run the tidal model for 240 days and used the TS of the last 183 days for HA when investigating eight major constituents (M_2, S_2, K_1, O_1, N_2, K_2, P_1, and Q_1) in the SCS. Rosenfeld et al. (2009) ran the tidal model for 56 days and performed tidal analysis on the last 34 days when studying semidiurnal and diurnal tides in central California. To study the eight major constituents at the Bass Strait, Wijeratne et al. (2012) ran the tidal model for a continuous period of 40 years (1970–2009) and used the yearlong data from 2003 to 2004 to perform HA. When simulating eight major constituents in Prince William Sound, Wang et al. (2013) found that the tidal amplitude and phase estimation with one-month-long outputs and those with one-year-long outputs were approximately the same.

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From all the works mentioned above, it can be found that when simulating four or more constituents, the length of TS for HA varies from about one month (e.g., 28, 29, 30, 34 days) to one year. So, the aim of this study is to seek the optimum length of TS for HA. In this paper, a two-dimensional tidal model is used to simulate the four principal constituents in the Bohai Sea (BS), the Bohai and Yellow Seas (BYS), and the Bohai, Yellow, and East China Seas (BYECS). A method is put forward to determine the optimum length of TS for HA with the generated surface elevation. The paper is organized as follows: The tidal model and settings are described in section 2. The method to determine the optimum length of TS for HA is described in section 3. Results and discussion are displayed in section 4. Finally, the paper is summarized in section 5.

2. Tidal model and settings

The governing equations used in this paper are the same as those in Lu and Zhang (2006), which are the vertically integrated equations of continuity and momentum:

$$\frac{\partial \zeta}{\partial t} + \frac{1}{a}\frac{\partial[(h+\zeta)u]}{\partial \lambda} + \frac{1}{a}\frac{\partial[(h+\zeta)v\cos\phi]}{\partial \phi} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + \frac{u}{a}\frac{\partial u}{\partial \lambda} + \frac{v}{R}\frac{\partial u}{\partial \phi} - \frac{uv\tan\phi}{R} - fv + \frac{cu\sqrt{u^2+v^2}}{h+\zeta} - A\Delta u + \frac{g}{a}\frac{\partial(\zeta-\overline{\zeta})}{\partial \lambda} = 0, \tag{2}$$

$$\frac{\partial v}{\partial t} + \frac{u}{a}\frac{\partial v}{\partial \lambda} + \frac{v}{R}\frac{\partial v}{\partial \phi} + \frac{u^2\tan\phi}{R} + fu + \frac{cv\sqrt{u^2+v^2}}{h+\zeta} - A\Delta v + \frac{g}{R}\frac{\partial(\zeta-\overline{\zeta})}{\partial \phi} = 0, \tag{3}$$

where t is the time; λ and φ are the longitude and latitude respectively; h is the undisturbed water depth; ζ is the sea surface elevation; $\overline{\zeta}$ is the tidal potential; u and v are the east and north components, respectively, of the fluid velocity; R is the radius of the earth; $a = R\cos\varphi$; g is the acceleration due to gravity; f is the Coriolis parameter; c is the bottom friction coefficient (BFC) of nonlinear bottom friction; A is horizontal eddy viscosity coefficient; and Δ is the Laplacian operator based on the spherical coordinate. The governing Eqs. (1)–(3) are discretized using the finite difference method on the Arakawa C grids. Refer to appendix A of Lu and Zhang (2006) for details.

The computed area is the BYS (34°–41°N, 117.5°–122.5°E shown in Fig. 1). The horizontal resolution

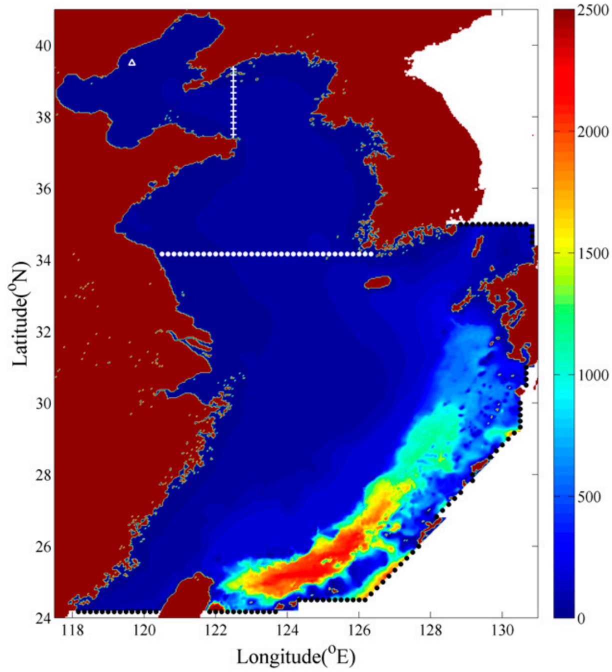

FIG. 1. Bathymetry of the BYECS. White pluses, and white and black dots indicate locations of the open boundary of the BS, the BYS, and the BYECS. Elevation data at the white triangle in the BS are used for spectra analysis.

is 1/6° × 1/6° and the time step is 60s according to the Courant–Friedrichs–Lewy (CFL) condition. The spatially varying parameterization scheme of the bottom friction coefficient is used, which is the same as that in Lu and Zhang (2006). The horizontal eddy viscosity coefficient is equal to $1500\,\mathrm{m^2\,s^{-1}}$. Initial conditions are such that the sea surface elevation and horizontal velocities are equal to zero. The model is forced by tidal potential and open boundary conditions. Tidal potential is calculated as Foreman et al. (1993). On the open boundary, the sea surface elevation can be expressed as

$$\zeta(t) = \sum_i \gamma_i H_i \cos(\omega_i t - \theta_i + \sigma_i + \delta_i), \tag{4}$$

where i represents the tidal constituent, H_i and θ_i are the HCs, ω_i is the angular speed, γ_i is the nodal factor, σ_i is the nodal angle, and δ_i is the initial phase. In this study, the four principal constituents (M_2, S_2, K_1, and O_1) are simulated simultaneously, of which the HCs on the open boundary are obtained from the Oregon State University Tidal Inversion Software (Egbert and Erofeeva 2002).

The model starts on 1 January 2000 and runs for a total of 1500 days. Simulated results of the whole-field surface elevation are recorded every single hour.

3. Methodology

To seek the optimum length of TS for HA, a method is put forward, which is described as follows:

1) Several nonoverlapping TS of the same length are chosen after the simulation becomes stable. Denote the total number of TS as M.
2) Each surface elevation TS is subject to HA to generate HCs of the four principal constituents. It should be noted that only the four principal constituents are considered in HA, and HA is carried out at all the computed grids in the area rather than at certain special grids. The HCs of the four constituents are written as $H_{i,j,k,m}$ and $\theta_{i,j,k,m}$, where i represents the tidal constituent, j and k are locations of computed grids, and $m = 1, 2, \ldots, M$. Because any two TS are nonoverlapping, differences exist between any two HC fields for each constituent.
3) The mean HC field corresponding to each constituent is calculated, which is defined as

$$\overline{H}_{i,j,k} = \frac{1}{M}\sum_{m=1}^{M} H_{i,j,k,m} \quad \text{and} \quad \overline{\theta}_{i,j,k} = \frac{1}{M}\sum_{m=1}^{M} \theta_{i,j,k,m}. \tag{5}$$

Next, the mean vector difference (MVD) between $(H_{i,j,k,m}, \theta_{i,j,k,m})$ and $(\overline{H}_{i,j,k}, \overline{\theta}_{i,j,k})$ is calculated:

$$\Delta r_m = \frac{1}{4N}\sum_{i=1}^{4}\sum_{j,k}\sqrt{(H_{i,j,k,m})^2 + (\overline{H}_{i,j,k})^2 - 2H_{i,j,k,m}\times\overline{H}_{i,j,k}\times\cos(\theta_{i,j,k,m} - \overline{\theta}_{i,j,k})}, \tag{6}$$

where N represents the total number of computed grids. Since M nonoverlapping TS are used, the average value of MVDs (AMVD) makes more sense, which is calculated as

$$\Delta r = \frac{1}{M}\sum_{m=1}^{M} \Delta r_m. \tag{7}$$

According to Eq. (7), the AMVD reflects differences of both amplitude and phase of all the constituents, which is a field-average deviation of HA results. It should be noted that the AMVD is a function of the length of TS.
4) Changing the length of TS and repeating processes 1–3 over a certain range (e.g., from 15 to 365 days), the variation of the AMVD can be obtained. Because the AMVD reflects the field-average deviation of HA results, when the AMVD is sufficiently small (smaller than a critical value), all the corresponding $H_{i,j,k,m}$ ($\theta_{i,j,k,m}$) are almost the same. In other words, the HA results do not depend on the choice of TS, and the corresponding length is regarded as the optimum.

4. Results and discussion

Based on the method mentioned above, the initial 30-day-long TS is discarded to make sure that the simulation is stable. According to Rayleigh criterion, to separate the four principal constituents, the minimum alias synodic period (shown in Table 1) is 14.8 days. Therefore, in this study, the length of TS varies from 15 to 365 days with an interval of 1 day. In addition, the total number of TS (M) varies with the length of TS, which is shown in Table 2.

Figure 2 displays the AMVD as a function of the length of TS. The MVDs corresponding to TS of various lengths are also plotted. On the whole, both the AMVD and MVDs first decrease and then increase as the length of TS increases, suggesting that a longer TS is not necessarily better. When the length of a TS is between 180 and 195 days, the AMVD is smaller than 0.075 cm, the critical value in this study, suggesting that the lengths ranging from 180 to 195 days are the optimum lengths of TS for HA in the simulation of the four principal constituents.

Next, it is necessary to investigate whether the optimum length changes when different computed area and model settings are used. As shown in Table 3, 10 more experiments (the previous one is regarded as E0) are carried out to investigate the effects of advection, bottom friction, nodal corrections, and computed area. In E1, E5, and E9, linear bottom friction in the Ekman form $(-c_E u/h, -c_E v/h)$ is used, in which the BFC $c_E = 1.2 \times 10^{-3}\,\mathrm{m\,s^{-1}}$. In E2 and E7, linear bottom friction in the Rayleigh form $(-c_R u, -c_R v)$ is used, in which the BFC $c_R = 1 \times 10^{-4}\,\mathrm{s^{-1}}$. In E4, nonlinear bottom friction is used and $c = 2.2 \times 10^{-3}$. In other experiments, the bottom friction is nonlinear and the BFC is the same as that in Lu and Zhang (2006). The AMVD in each experiment is calculated using the method mentioned in section 3.

Figure 3a displays the AMVDs in E0–E6, where the computed area is the BYS. Apparently, the AMVDs can be divided into two groups, that is, the linear and nonlinear bottom friction. When the linear bottom

TABLE 1. Alias synodic periods (days) of each pair of constituents. The boldface value shows the minimum alias synodic period to separate the four principal constituents.

	S_2	K_1	O_1
M_2	**14.8**	1.1	1.0
S_2		1.0	0.9
K_1			13.7

TABLE 2. Length of TS and the corresponding number.

Length of TS (days)	No. of TS
15–60	24
61–120	12
121–180	8
181–365	4

friction is used, the AMVD is always much smaller than the critical value (0.075 cm), suggesting that any length of TS larger than 15 days is suitable for HA. More importantly, this result has no relation with the form of the linear bottom friction and advection. In fact, the advection is much smaller than other terms in the momentum equations, which can be easily obtained by dimensional analysis. In addition, comparison of the spectra results in E1 and E5 (displayed in Figs. 4a,c) shows that the advection leads to no obvious peaks at higher harmonics, suggesting that the effect of advection can be neglected. When the nonlinear bottom friction is used (E0, E3, E4, and E6), the tendencies of AMVDs are almost the same, although different spatial distribution of BFC and nodal corrections are used, suggesting that the nonlinear bottom friction itself determines the trend of AMVD. Figure 3b shows the AMVDs in different computed areas (the BS, the BYS, and the BYECS). In E0, E8, and E10, although the magnitude of AMVDs differs, the general trend of all the AMVDs is almost the same, suggesting that the optimum length of TS for HA does not vary with computed areas. In E7 and E9, the AMVD is also much smaller than the critical value, which further indicates that any length of TS larger than 15 days is appropriate for HA when the linear bottom friction is used in the model.

Finally, why the optimum length of TS for HA is 180–195 days is investigated. As seen in Fig. 4, the nonlinear bottom friction results in the appearance of higher harmonics, whose frequencies and periods are listed in Table 4. However, because only the four principal constituents are considered in the HA, higher harmonics will lead to errors in the HA results of the M_2, S_2, K_1, and O_1 constituents, which can be easily tested by ideal experiments. Therefore, the surface elevation at every computed point can be rewritten as

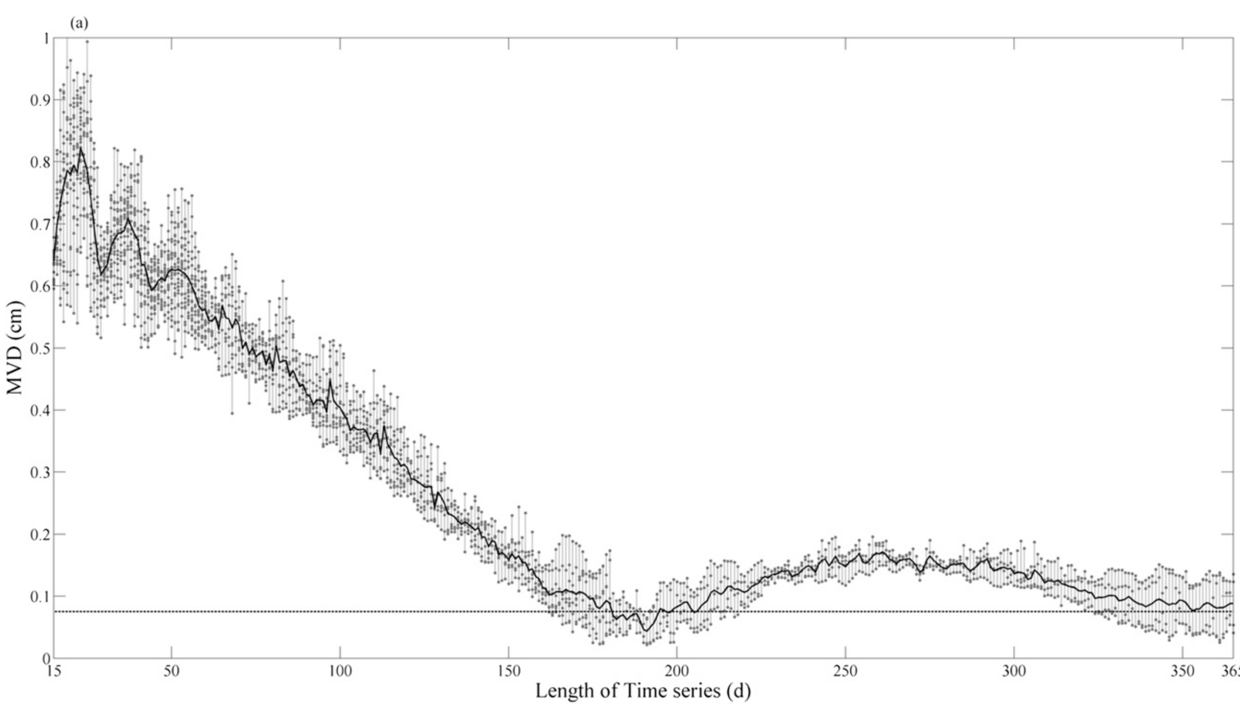

FIG. 2. MVDs (gray dots) and AMVD (black line) vs the length of TS. Dashed line represents the critical value (0.075 cm).

TABLE 3. Detailed settings of the 11 experiments.

Expt	Area	Advection	Bottom friction (BFC)	Starting time
E0	BYS	Yes	Nonlinear (Lu and Zhang)	1 Jan 2000
E1	BYS	No	Linear ($c_E = 1.2 \times 10^{-3}\,\mathrm{m\,s^{-1}}$)	1 Jan 2000
E2	BYS	No	Linear ($c_R = 1 \times 10^{-4}\,\mathrm{s^{-1}}$)	1 Jan 2000
E3	BYS	No	Nonlinear (Lu and Zhang)	1 Jan 2000
E4	BYS	No	Nonlinear ($c = 2.2 \times 10^{-3}$)	1 Jan 2000
E5	BYS	Yes	Linear ($c_E = 1.2 \times 10^{-3}\,\mathrm{m\,s^{-1}}$)	1 Jan 2000
E6	BYS	Yes	Nonlinear (Lu and Zhang)	1 Jan 1900
E7	BS	Yes	Linear ($c_R = 1 \times 10^{-4}\,\mathrm{s^{-1}}$)	1 Jan 2000
E8	BS	Yes	Nonlinear (Lu and Zhang)	1 Jan 2000
E9	BYECS	Yes	Linear ($c_E = 1.2 \times 10^{-3}\,\mathrm{m\,s^{-1}}$)	1 Jan 2000
E10	BYECS	Yes	Nonlinear (Lu and Zhang)	1 Jan 2000

$$\zeta(t) = S_0 + \sum_{i=1}^{4} \gamma_i H_i \cos(\omega_i t - \theta_i + \sigma_i + \delta_i) + \sum_{l=1}^{16} S_l, \quad (8)$$

where S_0 is the steady term and S_l ($l = 1, 2, \ldots, 16$) are errors caused by the higher harmonics, which are in the sinusoidal forms. When HA is carried out to calculate HCs of the M_2, S_2, K_1, and O_1 constituents using the least squares method, errors caused by S_l vary with the length of TS. When the length of TS is close to multiples of periods of each S_l, these errors will decrease. As seen in Table 4, except for M_2-S_2 and K_1-O_1, periods of other signals are all close to one, one-half, one-third, or one-fourth day, suggesting that any integer-day-long TS is close to multiples of periods of these signals. But for M_2-S_2 and K_1-O_1, whose periods are 14.8 and 13.7 days, respectively, in order to be close to multiples of periods of both of them, the length of TS should be in the range between 182 days (14 × 13 days) and 210 days (15 × 14 days). So, it is reasonable that the optimum length in simulating M_2, S_2, K_1, and O_1 constituents using the nonlinear bottom friction is 180–195 days. Therefore, it can be concluded that the nonlinear bottom friction leads to the appearance of higher harmonics

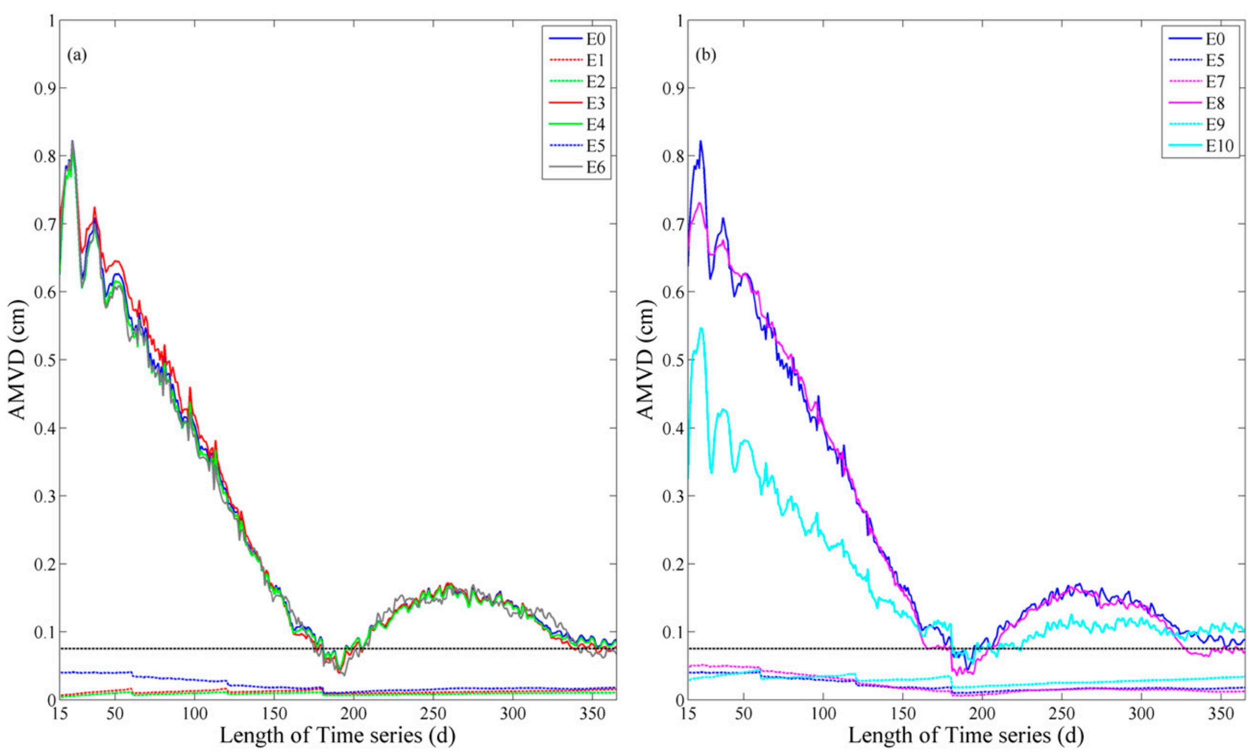

FIG. 3. AMVDs in (a) E0–E6, where the computed area is the BYS; and (b) different computed areas. Dashed line represents the critical value (0.075 cm).

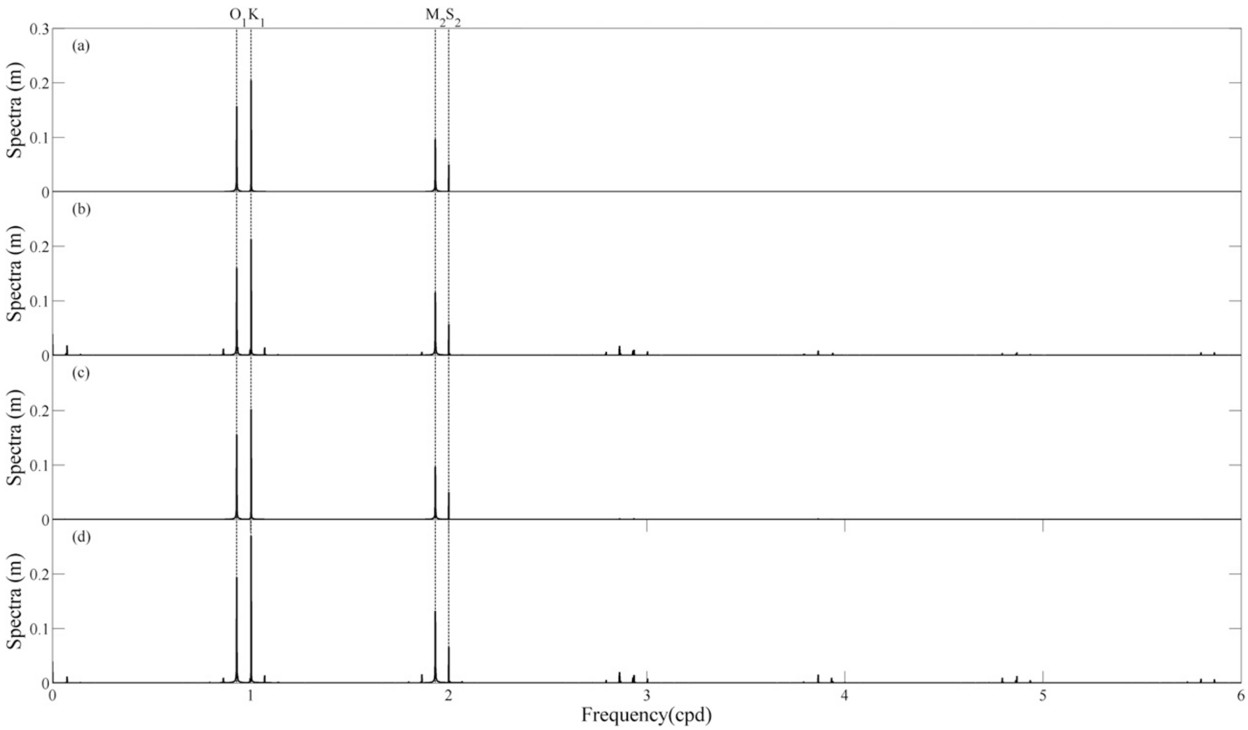

FIG. 4. Amplitude spectra of surface elevation at the station (white triangle in Fig. 1) in the BS for (a) E1, (b) E3, (c) E5, and (d) E0.

and thus the 180–195 days optimum length of TS for HA in the simulation of the four principal constituents.

5. Summary

For the simultaneous simulation of multiple constituents, the length of TS for HA varied from about one month to one year and its optimum value has not been investigated. Therefore, in this paper, a method is put forward to seek the optimum length of TS for HA. Based on the simulated results of the surface elevation, HA is carried out for several nonoverlapping TS of the same length, which varies from 15 to 365 days, and the AMVD is calculated. Because the AMVD reflects the field-average deviation of HA results, when the AMVD is sufficiently small, all the corresponding HA results are almost the same. In other words, the HA results do not depend on the choice of TS, and the corresponding length is regarded as the optimum. Results indicate that the range of 180–195 days is the optimum length of TS for HA in the simulation of the four principal constituents. To investigate what determines the optimum length, 10 more experiments with different computed area and model settings are carried out. Results indicate that the optimum length is independent of advection, nodal corrections, and computed area and only depends on bottom friction. When the linear bottom friction is used, any length of TS larger than 15 days is suitable for HA. However, the nonlinear bottom friction leads to the appearance of higher harmonics and thus the 180–195-day optimum length of TS for HA in the simulation of the four principal constituents.

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TABLE 4. Frequencies (s^{-1}) and periods (days) of the higher harmonics caused by the nonlinear bottom friction.

Frequency	Period	Frequency	Period	Frequency	Period	Frequency	Period
$2\omega_{M2}$	0.259	$\lvert\omega_{M2} - \omega_{K1}\rvert$	1.08	$\omega_{S2} + \omega_{K1}$	0.333	$2\omega_{K1}$	0.499
$\omega_{M2} + \omega_{S2}$	0.254	$\omega_{M2} + \omega_{O1}$	0.349	$\lvert\omega_{S2} - \omega_{K1}\rvert$	1.00	$\omega_{K1} + \omega_{O1}$	0.518
$\lvert\omega_{M2} - \omega_{S2}\rvert$	14.8	$\lvert\omega_{M2} - \omega_{O1}\rvert$	0.997	$\omega_{S2} + \omega_{O1}$	0.341	$\lvert\omega_{K1} - \omega_{O1}\rvert$	13.7
$\omega_{M2} + \omega_{K1}$	0.341	$2\omega_{S2}$	0.25	$\lvert\omega_{S2} - \omega_{O1}\rvert$	0.934	$2\omega_{O1}$	0.538

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REFERENCES

Cao, A. Z., Z. Guo, and X. Q. Lv, 2012: Inversion of two-dimensional tidal open boundary conditions of M_2 constituent in the Bohai and Yellow Seas. *Chin. J. Oceanol. Limnol.,* **30,** 868–875, doi:10.1007/s00343-012-1185-9.

Cummins, P. F., and L. Y. Oey, 1997: Simulation of barotropic and baroclinic tides off north British Columbia. *J. Phys. Oceanogr.,* **27,** 762–781, doi:10.1175/1520-0485(1997)027<0762:SOBABT>2.0.CO;2.

Egbert, G. D., and S. Y. Erofeeva, 2002: Efficient inverse modeling of barotropic ocean tides. *J. Atmos. Oceanic Technol.,* **19,** 183–204, doi:10.1175/1520-0426(2002)019<0183:EIMOBO>2.0.CO;2.

Fang, G. H., Y. K. Kwok, K. J. Yu, and Y. H. Zhu, 1999: Numerical simulation of principal tidal constituents in the South China Sea, Gulf of Tokin and Gulf of Thailand. *Cont. Shelf Res.,* **19,** 845–869, doi:10.1016/S0278-4343(99)00002-3.

Foreman, M. G. G., R. F. Henry, R. A. Walters, and V. A. Ballantyne, 1993: A finite element model for tides and resonance along the north coast of British Columbia. *J. Geophys. Res.,* **98,** 2509–2531, doi:10.1029/92JC02470.

He, R. Y., and R. H. Weisberg, 2002: Tides on the West Florida shelf. *J. Phys. Oceanogr.,* **32,** 3455–3473, doi:10.1175/1520-0485(2002)032<3455:TOTWFS>2.0.CO;2.

Kang, S. K., S. R. Lee, and H. J. Lie, 1998: Fine grid tidal modeling of the Yellow and East China Seas. *Cont. Shelf Res.,* **18,** 739–772, doi:10.1016/S0278-4343(98)00014-4.

Lu, X. Q., and J. C. Zhang, 2006: Numerical study on spatially varying bottom friction coefficient of a 2D tidal model with adjoint method. *Cont. Shelf Res.,* **26,** 1905–1923, doi:10.1016/j.csr.2006.06.007.

Rosenfeld, L., I. Shulman, M. Cook, J. Paduan, and L. Shulman, 2009: Methodology for a regional tidal model evaluation, with application to central California. *Deep-Sea Res. II,* **56,** 199–218, doi:10.1016/j.dsr2.2008.08.007.

Wang, X. C., and Coauthors, 2013: Modeling tides and their influence on the circulation in Prince William Sound, Alaska. *Cont. Shelf Res.,* **63,** S126–S137, doi:10.1016/j.csr.2012.08.016.

Wijeratne, E. M. S., C. B. Pattiaratchi, M. Eliot, and I. D. Haigh, 2012: Tidal characteristics in Bass Strait, south-east Australia. *Estuarine Coastal Shelf Sci.,* **114,** 156–165, doi:10.1016/j.ecss.2012.08.027.

Zhang, J. C., X. Q. Lu, P. Wang, and Y. P. Wang, 2011: Study on linear and nonlinear bottom friction parameterizations for regional tidal models using data assimilation. *Cont. Shelf Res.,* **31,** 555–573, doi:10.1016/j.csr.2010.12.011.

Zu, T. T., J. P. Gan, and S. Y. Erofeeva, 2008: Numerical study of the tide and tidal dynamics in the South China Sea. *Deep-Sea Res. I,* **55,** 137–154, doi:10.1016/j.dsr.2007.10.007.